Mathematics
Practice Exam
2024

## Essential Skills (1 hour 30 minutes)

Total marks - 50
Attempt ALL questions.
You may use a calculator.
To earn full marks, you must show your working in your answers.
State the units for your answer where appropriate.
Write your answers clearly in the spaces provided in this booklet. Additional space for answers is provided at the end of this booklet. If you use this space, you must clearly identify the question number you are attempting.

Use blue or black ink.

The roots of $\quad a x^{2}+b x+c=0 \quad$ are $\quad x=\frac{-b \pm \sqrt{\left(b^{2}-4 a c\right)}}{2 a}$

Sine rule

$$
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}
$$

Cosine rule

$$
a^{2}=b^{2}+c^{2}-2 b c \cos A \quad \text { or } \quad \cos A=\frac{b^{2}+c^{2}-a^{2}}{2 b c}
$$

Area of a triangle
$A=\frac{1}{2} a b \sin C$

Volume of a sphere $\quad V=\frac{4}{3} \pi r^{3}$

Volume of a cone $\quad V=\frac{1}{3} \pi r^{2} h$

Volume of a pyramid

$$
V=\frac{1}{3} A h
$$

Standard deviation: $\quad s=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}$
or $s=\sqrt{\frac{\sum x^{2}-\frac{\left(\sum x\right)^{2}}{n}}{n-1}}$, where $n$ is the sample size.

## Total marks - 50

## Attempt ALL questions

1. The population of Glasgow is growing by $0.6 \%$ each year.

In 2022 the population was 1,689,000.
What is the projected population in 2025 ?
Give your answer to 4 significant figures.
2. The weight of the much maligned Scottish Midge is $5 \times 10^{-7}$ grams. The average adult human weight is 62 kilograms.
How many times heavier is the average adult to the average midge? Give your answer in scientific notation.
3. Expand and simplify $(x-3)\left(2 x^{2}-3 x+1\right)$.
4. S4 pupils in a school who attend morning supported study are offered free breakfast.

One week the order of 7 waffles and 6 breakfast rolls was billed to the school at £15.55.
(a) Write an equation to illustrate this information.
(b) Another week, 6 waffles and 5 breakfast rolls cost $£ 13 \cdot 10$.

Write an equation to illustrate this information.
(c) Calculate the cost of one waffle and one breakfast roll.
5. In the triangle ABC

- $B C$ is 7.3 centimetres.
- Angle BAC $=62^{\circ}$
- Angle $\mathrm{ACB}=39^{\circ}$


Calculate the length of side AC.
6. Express $\frac{3}{2 x-1} \div \frac{x+4}{4 x^{2}-1}, x \neq \pm \frac{1}{2}$, as a fraction in its simplest form
7. Solve the equation $5 \sin x^{\circ}+3=1,0 \leq x<360$.
8. Find $|\boldsymbol{u}|$, the magnitude of vector $\boldsymbol{u}=\left(\begin{array}{c}-8 \\ 12 \\ -24\end{array}\right)$
9. The prices, in pounds, of some flights to Germany from Scotland during the Euros are:

$$
\begin{array}{llllll}
87 & 101 & 91 & 144 & 127 & 134
\end{array}
$$

(a) Calculate:
(i) the mean.
(ii) the standard deviation.
(b) The mean price to the same destinations from England is $£ 83$ and the standard deviation is 32.5

Make two valid comparisons about the prices of flying to Germany from Scotland and from England.
10. The RNLI has 2 lifeboats based at the Boathouse in Anstruther.

After an SOS call in the Firth of Forth the lifeboats depart from the Boathouse, B.
The smaller lifeboat, the Akira (A), travels 15 km on a bearing of $062^{\circ}$.
The larger lifeboat, the Robert and Catherine Steen (S), travels 24 km on a bearing of $108^{\circ}$.


Calculate the distance between the two lifeboats.
11. An ice cream cone keyring is the shape of a cone with a hemisphere on top.

The keyring has

- Diameter 18 millimetres
- Height 34 millimetres

Give your answer to 2 significant figures.
12. The diagram shows the sector ACB.

The radius of the sector is 6.25 centimetres.


Calculate the area of the sector.
13. The diagram shows the circular cross section of a pipe with oil flowing through.

- O represents the centre of the circle
- $A B$ is the surface of the oil.
- The radius is $r$ centimetres.
- $A B$ is 16 centimetres.
- The depth of the oil at its deepest point is 4 centimetres.
- The distance from centre $O$ to the surface of the oil is $x$ centimetres.

(a) Write down an expression for r in terms of $x$.
(b) Calculate, $r$, the radius of the pipe.

